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Application of Symmetry Analysis to Dynamical Systems  
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### Final Report

#### Abstract

Group theoretical approach to study of symmetry properties, local conservation laws and inverse problem of variations is applied for a wide class of nonlinear partial differential equations. For the equations of the class the correspondence between symmetries and local conserved currents is established. Many interesting equations belong to the class, e.g. regularized long-wave equation, nonlinear diffusion equation and Navier-Stokes equations. A number of important differential identities was derived and shown to determine symmetry-related characteristics of differential systems.

The grant "Application of Symmetry Analysis to Dynamical Systems" (June 1, 1992 - November 15, 1996) was devoted to the study of the group properties, local conservation laws and related characteristics of partial nonlinear differential systems.

At the first stage of our study it was demonstrated for Lagrangian systems that non-Noether symmetry transformations for which the change of action functional  $S$  is proportional to  $S$  do not lead to any new local conservation laws different from Noether ones, and these non-Noether symmetries lead to non-local conservation laws [1]. Further the independent proof of more general results: for any Lagrangian system the set of local conservation laws can be covered by currents corresponding to Noether symmetries [2].

For differential systems without well-defined Lagrangian function, more general relations between symmetries, conservation laws and variational problems were shown to hold [2]. However for a large class of partial differential equations it is still possible to associate symmetries and conservation laws in natural way. This class includes such known equations as Korteweg-de Vries, modified KDV, Boussinesq equation, nonlinear heat equation, nonlinear wave equation, Kadomtsev-Petviashvili equation, nonlinear diffusion equation, regularized long-wave equation, Monge-Ampere equation, Euler equation, Navier-Stokes equations and many other equations. For this class of equations it is possible to calculate the characteristics of conservation laws via corresponding symmetry characteristics. For Lagrangian systems the approach leads to results consistent with the standard Noether theory. For non-Lagrangian systems the approach establishes the correspondence between the symmetries of differential systems and its local conservation laws.

At a later stage of our study a number of important operator relations were derived and connections with the generalized Helmholtz conditions and the inverse problem of variational calculus were investigated [3,4]. Among the differential identities studied is the identity based on the non-commutation of the canonical symmetry operator with the Euler-Lagrange operator, as well as related identities which play an important role in the analysis of Noether and non-Noether symmetries and in the formulation of equivalent Lagrangians of the problem. The key role of Noether symmetries is discussed and Noether conditions for the important case of contact transformations are obtained. Relations between equivalent Lagrangians and Noether symmetries are discussed in details. Non-Noether symmetries are shown to play a crucial role in generation of alternative Lagrangians. Equivalent and s-equivalent Lagrangians, connection with symmetries, and a role of special semi-Noether transformations are discussed [5].

One of our main goals was to establish the correspondence between symmetries of a differential equation and locally conserved currents of a described dynamical system in such a way that the approach is applicable for many physically interesting equations for which there is no known Lagrangian. The approach based on the Noether relation [2] associates every generalized symmetry operator with a corresponding set of conserved vectors. However, there is a limit of the applicability of the approach outlined in [2]. It was shown in [4] how to enlarge the class of differential equations for which conservation laws can be found using the procedure developed in [1]. This class was also shown to contain all differential equations, which are symmetry transformation of the original system [2]. It was also shown in [4] that it is possible to relate symmetries and conserved vectors for a quite general physical system and a formula for the characteristics of the corresponding conservation laws was derived.

The approach developed in [1-5] allows one to find conservation laws and related characteristics of many known equations of physical interest and plays a special role for the study of systems without well-defined Lagrangian function. The availability of special software programs for finding symmetries of differential equations (written on MATHEMATICA and MACSYMA) make this approach attractive for practical applications.

Finally, the class of differential equations with an infinite group of symmetry determined through one or more arbitrary functions was considered. For these equations via use of the second Noether theorem symmetry of the system with assumed behavior of solutions on the boundary, it turns out to be possible to write differential "constraints" defining the form of considered solutions. This procedure was applied to Navier-Stokes equations.

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